



INTERNATIONAL BLACK SEA UNIVERSITY
FACULTY OF COMPUTER TECHNOLOGIES AND ENGINEERING
PhD. PROGRAM

DEVELOPING OF A METHOD OF SYNTHESIS OF MULTILoop LC-CIRCUITS
WITH PREDEFINED RESONANCE FREQUENCIES

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Extended Abstract of Doctoral Dissertation in Computer Science

Tbilisi, 2016

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Introduction

Knowledge and understanding of the full spectrum of eigen values is especially important in the development of technical systems of different nature: electrical, mechanical, electro-mechanical and others. The new method of designing (synthesis) multiloop LC-circuits with predefined values of resonance frequencies (eigen values of characteristic polynomials) has been developed; The conditions when predefined resonance frequencies are available have been determined; Necessary software to realize synthesis of multiloop LC-circuits with predefined values of resonance frequencies has been created; Various types of multiloop LC-circuits have been synthesized.

Analysis and synthesis of resonance and anti-resonance frequencies of the multi loop LC electric circuits has been key studies in the field of engineering and industrial technology. Outputs generated through passive electric circuits contain significant and valuable information for the operation of the systems and complex apparatus. Since it is vital, they require deep analytical observations and calculations in order to reach productive, effective, exact conclusions.

On the other hand understanding eigen values and eigen vectors of the full spectrum or multi loop LC circuits gives clear information to determine precise results and values to benefit from in the case of important situations (Kron, 1959) and (Skudrzuk, 1971). In this study various examples are taken to consider their individual eigen values which are used as an essential inputs of the system ultimately to synthesize results in terms of expected values are exist or not. These steps can not be achieved simply by just analysis of transfer function issues, rather they need tough investigations of historical background of electrical phenomena that the consequences of the research should contribute and help to make further studies with more rich and potentially clear knowledge.

Knowing that LC elements are crucial part of this study, from simpler to complex circuits are considered with several trials to develop and understand the core purpose of the research thesis. For instance, assessment of one circuit is based on couple of steps from assigning all branch elements with all unlikely impedance values up to group of similar impedances. Finally, relying on the theories stated throughout the study, generated outputs illustrate remarkable and significant results, where initially defined values are still conservative if required criteria are obeyed. Furthermore, the results can be good example and tool for the practicality of the research that the conservative values in other words angular frequencies of each branch element in the

circuit are frequencies, which can be allowable band pass area for signals to pick up, hold and exploit from them.

Another importance of the understanding and analysis of the multi loop LC circuits is their capability of working not only for low frequency responses but for higher frequencies as well. Study widely stated to the examples of with different kind of possibilities in terms of functionality.

One more worth considering issue about the study is unquestioningly to find solution for the problems that everyday engineering field confront with. Careful analysis on the LC elements would help to overcome problems such as damping, irregular oscillations and of course to avoid from possible dangers.

The current study focuses on the development of the method of analysis and synthesis of LC multi loop circuits with predefined eigen values. In other words these eigen values are the roots of the loops in the circuit which are angular frequencies and frequency response of the system. To receive initially assigned values from the final state is our conservative values those we are interested in throughout of the study.

Structure of the Thesis

Chapter I is Literature review of relevant studies, generally encompasses inductive and capacitive researches with their applications, contributions in the engineering field and how they are crucial part of the most electrical and new technological inventions and developments. Among the studies done before oscillatory systems with LC elements, n-dimensional multiloop circuits, potential background of geometric approaches of the engineering systems are taken into considerations.

Overall Chapter II is about theoretical and mathematical computations of the study. Definition of the problem is done. Overall large dimensional network circuits are mentioned with their parameters, possible outputs and values. Also pure node, pure loop circuits are taken as the basis of the calculations. Lastly, important propositions are stated in order to reach to the results.

Empirical calculations are done in Chapter III, where various types of multiloop LC circuits with their graphical and parametric definitions are given. Based on the propositions (1 to 10) particularly propositions 9, and 10 are taken into account, in which conservative eigen values are underlined as a key subject of the thesis in the analysis and synthesis of the circuits.

Methodology

Based on the electric circuit theorems and analysis methods, tensorial and linear algebra for the geometric approaches of the multiloop electrical circuits, MatLab programming language is used to develop required algorithmic computations. Higher order polynomials are created while calculating higher order or multi-dimensional circuits. Thus, conditions of available predefined eigen values are determined under the light of Kron's tensor theory of electrical networks (circuits), Weinstein and Aronszajn's Intermediate problem's method. Where G. Kron introduces three kinds of circuits; pure loop, pure node and orthogonal. The first two kinds of circuits – pure-loop and pure-node – are not considered in the traditional circuit theory. G. Kron calls ordinary circuits orthogonal. A pure-loop circuit is defined as a circuit in which the number of independent loops is equal to the number of branches or to the number of impedances, i.e. $k=n$ and $m=0$. Pure-loop circuits are not considered in the classical circuit theory constructed by the graph theory where it is always assumed that $n>k$.

A pure-node circuit is defined as a circuit in which the number of independent node pairs is equal to the number of branches or to the number of impedance s, i.e. $m-1 = n$ and $k = 0$. In the classical theory these circuits are not considered either as they do not contain closed loops (this is exactly what the equality $k = 0$ means).

Orthogonal circuits are ordinary circuits for which $k \neq 0$ and $m \neq 0$ and the Euler relation $k + m-1 = n$ holds (the latter relation is, of course, also fulfilled for pure-loop and pure-node circuits).

Ultimately stated propositions define objectives and results of the research through associating with the theories mentioned above.

Objectives of the Research

The analysis of relevant publications allowed to determine the following objectives of the research:

- Developing a new method of designing (synthesis) multiloop LC-circuits with predefined values of resonance frequencies (eigen values of characteristic polynomials);
- Determining the conditions when predefined resonance frequencies are available;
- Creating necessary software to realize synthesis of multiloop LC-circuits with predefined values of resonance frequencies;
- Applying the developed method for various types of multiloop LC-circuits.

Novelties and Contributions

- The new methodology of developing of a synthesis of multi loop LC circuits was elaborated;
- Appropriate software tools (in MATLAB programming language) were created;
- Based on the propositions stated in theoretical foundations, various conditions for the *n-dimensional circuits* was experienced and applied to get concrete results;
- Pure-node and pure-loop circuit impedances were taken into considerations;
- The consequences mentioned above allowed creating the new principles of analysis and calculation of multiloop circuit topologies.
- On the base of the mentioned methods, effective propositions were stated if available conditions are obeyed eigen values of the pure loop circuits are conservative

Structure and Volume of the Work

The thesis study is 113 pages and consists of 3 chapters, list of references, list of figures and list of tables.

Scientific and Practical Importance

In the thesis study, to compute resonance frequency and conservative eigen values of the *n-dimensional* or multiloop circuits;

1. Loop matrices of inductances L^k and capacitances defined by the following transformations

$$L^k = \Gamma L^d \Gamma'$$

$$C_k = \Gamma C_d \Gamma'$$

In this case, *k*-order loop matrix of impedances is equal to

$$Z^k = \lambda L_k - C_k,$$

Where λ - eigenvalues of the generalized problem squares are equal to their own angular frequencies $\lambda = \omega^2$ of circuits.

2. Loop-Branch matrix table is established

3. Special Matlab program is run using data from the Loop-Branch matrices and impedance values located on each individual serially connected inductors and capacitors.
4. Finally generated outputs are analyzed and synthesized to determine whether there occurs conservative eigen values or not, in other words predefined eigen values.
5. Developments under the light of current study enables further studies to integrate onto more sophisticated systems relevant with the areas such as signals, DSP, filters etc.

CHAPTER 1: LITERATURE REVIEW

RLC passive elements are used in variety fields of engineering technology, from simple on-off circuits until industrial complex systems. Especially in our daily life, we confront with them at every single level of electric and electronic devices that we benefit from. These elements actually are just two terminal devices but their contribution to the engineering plays significant key roles in the designing of the circuits and systems.

Electric circuits exemplify oscillatory systems with concentrated parameters (Skudrzuk, 1971) and (Kron, 1959). The problem on their eigenvalues and eigenvectors is, on the one hand, one of the most vital issues of mathematical physics and the oscillation theory, and, on the other hand, general since it has many different applications. The characteristic feature of systems with concentrated parameters is that they possess a geometric structure – a graph (or a simplicial complex if we speak in terms of algebraic topology). Eigen values or in engineering view point resonance frequencies of the systems are key parameters in order to design and model electric systems. If available parameters of resonance frequencies are computed, different types of electrical, electro mechanical, oscillatory and mixed systems can be built. As a result, possible dangers, hazardous situations can be avoided.

CHAPTER 2: DEFINITION AND THEORY OF THE PROBLEM

Tensor Theory of Electrical Circuits

Three types of Electrical Circuits

1. PURE-LOOP
2. PURE-NODE
3. ORTHOGONAL

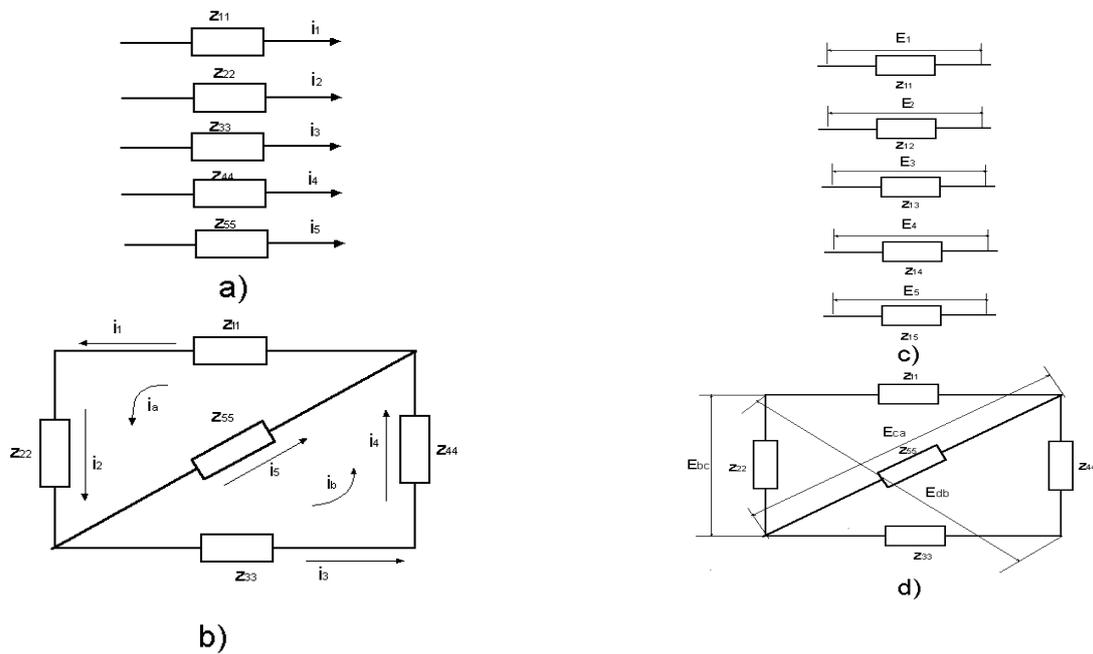
A **pure-loop** circuit is defined as a circuit in which the number of independent loops is equal to the number of branches or to the number of impedances, i.e. $k=n$ and $m=0$.

A **pure-node** circuit is defined as a circuit in which the number of independent node pairs is equal to the number of branches or to the number of impedances, i.e. $m-1 = n$ and $k = 0$.

The **primitive** circuit is simply a set of five impedances not connected with each other. To each of these impedances are formally assigned the values of current i and voltage e .

Orthogonal circuits are ordinary circuits for which $k \neq 0$ and $m \neq 0$ and the Euler relation $k + m - 1 = n$ holds (the latter relation is, of course, also fulfilled for pure-loop and pure-node circuits).

Orthogonal and Primitive Circuits



Transformations from Primitive to Orthogonal Circuits;

$$i = Ci'$$

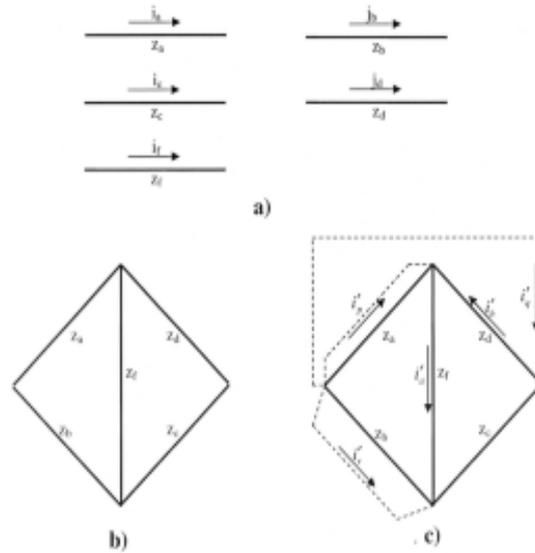
Invariants of Power;

$$ei = e'i'$$

Transformations of Impedance Matrices;

$$Z' = C^T Z_D C$$

Transformation Matrix C is singular (non square). Pure-loop and Primitive Circuits;



The dotted lines show fictitious loops. Transformations from Primitive to Pure-Loop Circuits;

$$Z^{(5)} = C^T Z_D C$$

Matrix C is not singular and it has inverse;

$$Z_D = (C^{-1})^T Z^{(n)} C^{-1}$$

All pure-loop circuits of the initial primitive circuit possess pairwise equal eigenvalues equal in their turn to the eigenvalues of the primitive circuit. Basic problem is pure-loop circuit. By imposing constraints on fictitious loops a finite recurrent process from basic pure-loop circuit to initial orthogonal circuit can be constructed. Imposing of one constraint implies. The operator $Z^{(n-1)}$ which is a part of the operator $Z^{(n)}$ and defined on the subspace L^{n-1} is represented in the coordinate from as a principal submatrix of order n-1 of the matrix $Z(n)$.

$$Z^{(n)}(\lambda), Z^{(n-1)}(\lambda), \dots, Z^{(n-i)}(\lambda), \dots, Z^{(k)}(\lambda)$$

where $\lambda = \omega^2$.

Eigenvalues of the base oscillatory system are called eigenvalues of zeroth order (they correspond to the operator, while eigenvalues obtained by imposing i constraints on a pure loop circuit (they correspond to the operator $Z^{(n-i)}(\lambda)$), are called eigenvalues of i-th order. Thus to each k-loop circuit there correspond n-k series of eigenvalues

$$\lambda_1^{(0)}, \dots, \lambda_n^{(0)}; \lambda_1^{(1)}, \dots, \lambda_{n-1}^{(1)}; \dots; \lambda_1^{(n-k)}, \dots, \lambda_k^{(n-k)}$$

An eigenvalue of zeroth order is called *i*-conservative if it is preserved when *i* constraints are imposed and vanishes when *i* + 1 constraints are imposed. An eigenvalue of zeroth order is called conservative if it is preserved when *n*-*k* constraints are imposed, i.e. it is an eigenvalue of both a pure loop circuit and a *k*-loop circuit.

Basic Theorem

Eigenvalues of zeroth order of a pure loop circuit (of a base oscillatory system) are conservative either the impedances of all principal branches are equal (where *k* – number of independent loops) or, the number *r* of equal impedances of any branches is greater than the number of node pairs (*m*=*n*-*k*-1) in a *k*-loop circuit.

Let us consider the same circuit as above but this time in terms of node analysis. Since the number of branches is 5, and the number of loops is 2, we have three node pairs (Figure 1. d). The role of vectors is now played by node voltages. Applying the second law of Kirchhoff, we obtain the following relations between the variables of primitive and node circuits;

$$\begin{pmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \end{pmatrix} = \begin{pmatrix} -1 & 0 & -1 \\ 0 & 0 & 1 \\ 0 & -1 & -1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} E_{cd} \\ E_{db} \\ E_{bc} \end{pmatrix},$$

or in the considered matrix form

$$E = AE' \tag{3}$$

In this case, from the geometric viewpoint, the transformation matrix *A* is also the Jacobian of transformation from the variables *E'* to the variables *E*.

The introduced notions associated with transformations of variables of a circuit are geometric by nature. It is obvious that for these transformations, impedance matrices and other objects of a circuit will change too. As an invariant of such transformations power of the circuit is used.

$$P = ei \tag{4}$$

where *i* and *e* are *n*-dimensional vectors. The power invariance implies that

$$ei = e'i' \tag{5}$$

where i and e are n -dimensional vectors of currents and voltages of the primitive circuit; i' and e' are n -dimensional vectors of currents and voltages of the connected circuit. In terms of geometry, this equality means that the transformation of bases (the matrix C in (2)), which preserves the scalar product (5).

The power invariant enables us to derive transformation formulas for voltages and for an impedance matrix. Let an equation of a primitive circuit have the matrix form

$$e = Z_D i \quad (6)$$

where Z_D is the diagonal matrix of impedances of primitive circuit.

Currents of connected and primitive circuits are related by (2). If in (4) we replace the vector i by the right-hand side of (2), then

$$P = ei = eCi'$$

On the other hand, from (5) we have

$$e'i' = eCi', \text{ hence}$$

$$e' = eC = C^T e$$

The latter equality implies that impact voltages of the circuit (in loop analysis) are transformed by means of the matrix transposed with respect to the current transforming matrix C . Using the above-mentioned postulate on power invariance, it is likewise easy to obtain transformation formulas for the impedance matrix.

If in the matrix equation (6) of a primitive circuit the vector i is replaced by vector Ci' , then

$$e = Z_D Ci'$$

Multiplying both parts of the latter equation by matrix C^T , we obtain

$$C^T e = C^T Z_D Ci'$$

But since $C^T e = e'$, we have

$$e' = C^T Z_D Ci'$$

which implies

$$Z' = C^T Z_D C \quad (7)$$

where Z' is $k \times k$ impedance matrix of connected circuit.

Formula (7) transforms the impedance matrix of the primitive circuit to the impedance matrix of the connected circuit when the basis transformation (2) is used. We have thus obtained a complete picture of transformations of the objects of a primitive circuit to the objects of the connected circuit.

Quite analogously, we can repeat this process for node variables as well. We omit the derivation of these relations as they literally repeat the above reasoning and immediately yield the final result.

Basis transformations for node variables are given by (3). Transformations of node currents (external disturbances) are defined as

$$I' = A^T I \quad (8)$$

while the conductance matrix is transformed by

$$Y' = A^T Y_D A \quad (9)$$

where Y_D is the diagonal conductance matrix of the circuit.

The algebraic structure of (2), (6), (7), on the one hand, and that of (3), (8), (9), on the other hand, are absolutely the same. The only difference is that another transformation matrix (another transformation Jacobian) and other space dimensions (k and $n-k$, respectively) are used. It is not difficult to verify that between the matrices C and A there exists the relation.

$$A^T C = 0 \quad (10)$$

The basic principles of approach to the representation of a circuit as a tensor-geometric object can be now formulated as follows. A circuit consisting of n elements is an n -dimensional geometric object described by the tensor equation

$$e = Zi \text{ (loop analysis)} \quad (11)$$

or by

$$I = YE \text{ (node analysis)}. \quad (12)$$

The concrete structure of a circuit depends on a choice of a basis, where tensor equations (11) or (12) take the concrete form. Moreover, the loop variables which form a k -dimensional subspace of the initial n dimensional space are transformed by means of the matrix C , while the node variables which form an $n-k$ -dimensional subspace – by means of the matrix A . Respectively, the k -dimensional impedance tensor Z is transformed by means of (7), and the $n-k$ -dimensional tensor Y – by means of (8). All transformations are such that a primitive circuit is considered as some circuit (true, it has no physical counterpart) which is characterized by the fact that, firstly, both the impedance matrix and the conductance matrix are n -dimensional and diagonal, and, secondly, both loop and node variables form n -dimensional spaces.

From the above reasoning it follows that the designing of all possible circuits from the initial set of n elements (n branches + their n impedance s) is actually reduced to the transformation of bases. In that case, from the geometric viewpoint, we deal with one and the same tensor object though describable in different bases (the latter circumstance is due to power invariance), while, from the engineering viewpoint, which is more particular, each concrete circuit is considered as an independently existing object.

CHAPTER 3: EXAMPLES AND APPLICATIONS

Multiloop Circuit

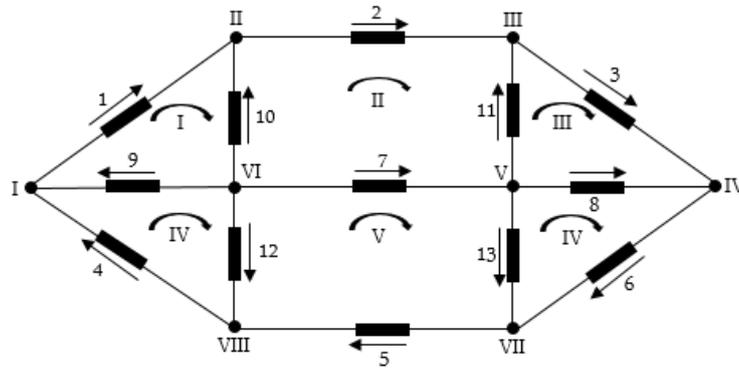


Figure 2. Electric circuit with 6 loops, 8 nodes and 13 branches

According to the circuit drawn in Figure 2 we will have maximum six different outputs of resonance frequency or eigen values of each loop in the network. However, the values depend on the assignment of the branches. The values span from minimum branch values till maximum one. If circuit is constructed under the condition (13) we have 2 cases

$$k \leq m - 1 \tag{13}$$

Where: k number of loops, m number of nodes

Case 1.

$$r \leq m - 1$$

Where: r number of identical branch impedances

Result: possible number of predefined resonance frequencies will definitely be 1.

Case 2.

$$r > m$$

Result: number of predefined resonance frequencies will rely on the equation (14)

$$r - (m - 1) \tag{14}$$

If circuit is constructed under the condition as in (15), we will have only one case in which result will depend on the equation (16).

$$k > m \tag{15}$$

Case 1.

$$r - (m - 1) \tag{16}$$

Assigning LC values;

In this case we will consider that number of similar branches are bigger than number of nodes of the circuit. Therefore, it should generate more than one predefined value from the output as a resonance frequency of the loop or eigen value of the certain loop. We preferred nine branches with equal values. Going deeper through the equation in which if the number of equal branches are bigger than the number of the nodes following equation will give the exact value;

$$Q = r - (m - 1)$$

Where r- number of identical branches and m - number of nodes of the circuit

➤ $r = 9$

➤ $m = 8$

➤ $Q = 2$

Two branch values will be seen from the output as an eigen value of the loops similar with their initially assigned values.

Circuit LC values of separate branches

Table 1

Branches	Inductances and Capacitances of Separated Branches												
	1	2	3	4	5	6	7	8	9	10	11	12	13
Inductance	2	2	2	2	2	2	2	2	2	10	11	12	13
Capacitance	4	4	4	4	4	4	4	4	4	100	121	144	169

Circuit Loop eigen values

Table 2

	Loops					
	1	2	3	4	5	6
Eigen values (ω^2)	2	2	7.7	9.5	10.3	11.7
Angular Frequency (ω)	1.41	1.41	2.78	3.09	3.21	3.43

Among the eigen values in Table 2, the number 2 is conservative with multiplicity of two which is even quantity. Therefore graph of the conservative eigen value 2 deflects horizontal line without crossing but just with smooth touch and then changes direction as in Figure 3. Worth noticing case, the deflection point in Figure 3 is the square root of the conservative eigen value which is angular frequency of the loop.

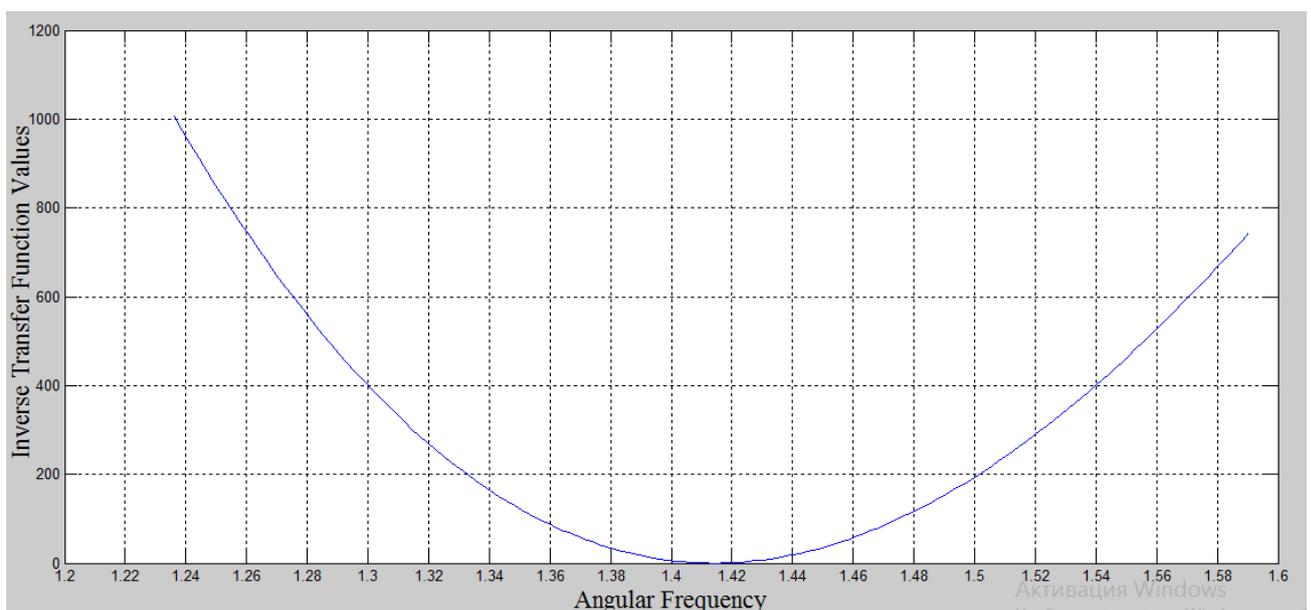


Figure 3. Fragment of Figure 2: area close to conservative eigen value 1.41

Definition 1

An eigenvalue of zeroth order $\lambda_j^{(0)}$ is called i-conservative if it is preserved when i constraints are imposed and vanishes when i + 1 constraints are imposed. An eigenvalue of zeroth order $\lambda_j^{(0)}$ is called conservative if it is preserved when n-k constraints are imposed, i.e. it is an eigenvalue of both a pure loop circuit and a k-loop circuit.

A corollary of this definition is;

Proposition 1

Eigenvalues of zeroth order of a pure loop circuit (of a base oscillatory system), the multiplicity r of which is greater than the number of node pairs in a k-loop circuit, are conservative.

However, the above proved proposition does not cover all possible cases, of zero order eigenvalues conservatively: it is not difficult to show that in the case when the number of independent loops is less than or equal to the number of node pairs, $k \leq m-1$, i.e. when the conditions of Proposition 2.11 is not met, selection of the same values of inductances and capacitances ($L_1 = L_1 = L_2 = \dots = L_k$ and $C_1 = C_2 = \dots = C_k$) for the impedances of all principal branches causes that the corresponding eigenvalue of zero order turns to be conservative. The latter follows from the structure of the loop matrix Z^k of the k-loop chain.

Indeed, consider the chain of the n branches and k loops. Let's denote the equal zero order eigenvalues of principal k branches as $\lambda_m^{(0)} = 1/L_i C_i$ ($i=1,2,\dots,k$). Further, let's enumerate principal branches of the circuit from 1 to k and consider the structure of an arbitrary row of the loop matrix Z^k . Each entry of the i-th row of the matrix Z^k is equal to either the negative value of the impedances of not principal branches bounded the i-th loop, or zero. The entry of the row which is located in the main diagonal of the Z^k is the sum of the impedances of branches of the i-th loop, including the impedance of the i-th principal branch. It follows that;

$$\det(Z^k(\lambda_m^{(0)}))=0,$$

Since the impedances of all principal branches included in the diagonal entries of the matrix Z^k , are equal to zero, and therefore the sum of all the columns of the matrix is zero, which implies the latter equality.

Proposition 2

Eigenvalues of zeroth order of a pure loop circuit (of a base oscillatory system) are conservative either the impedances of all principal branches are equal ($L_1 = L_1 = L_2 = \dots = L_k$ and $C_1 = C_2 = \dots = C_k$, where k – number of independent loops) or, the number r of equal impedances of any branches is greater than the number of node pairs ($m=n-k-1$) in a k -loop circuit.

The latter proposition proves to be rather effective, since conditions can be used in the synthesis of circuits with a predefined range of Eigen frequencies by simply choosing the required number of elements (impedances) of the same kind in a primitive circuit.

The above examples clearly depict crucial achievements and results for the statements stated throughout the study. Under the light of the theories mentioned above, any type of n -dimensional serially connected inductor and capacitor LC circuits with n - branches can be analyzed to compute frequency response characteristics by understanding eigen values and eigen vectors of the node, loop phenomena. Study substantiates that development of relevant methods for the analysis of LC circuits can contain remarkably huge key impacts for the openings such as filtering, admittance and possibility of danger avoidance issues in the field of electrical engineering.

Conclusions

(Results of the research)

Following results have been reached in terms of the objectives, novelties, contributions of the research; analysis, synthesis and development of the multiloop LC circuits.

1. The new method of designing (synthesis) multiloop LC-circuits with predefined values of resonance frequencies (eigen values of characteristic polynomials) has been developed;

2. The conditions when predefined resonance frequencies are available have been determined;
3. Necessary software to realize synthesis of multiloop LC-circuits with predefined values of resonance frequencies has been created;
4. Various types of multiloop LC-circuits have been synthesized.

Publications

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4. Duisheev. K. M, (2016). Developing a New Method of Designing (Synthesis) Multiloop LC-Circuits with Predefined Values of Resonance Frequencies. Science and Technology-Conference of Youth Scientists, Candidates and Students.